MATHEMATICS LEARNING CASE STUDY MODULE
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Overview of Mathematics Learning Case Study Module

This module focuses on student learning, and identity and dispositions of a case study student. In working with the case study student over the course of the semester, the prospective teacher PST considers how to use this knowledge in mathematics instruction. The activities within this module allow for PSTs to (a) conduct a mathematics "getting to know you" interview with a single student, (b) shadow a student for all or part of a day, (c) conduct one or more problem solving interview assessments with that same student, and (d) engage in written analysis and reflection on the first three activities in the module.

Goals for PSTs:

The goal of the Mathematics Learning Case Study Module is to help PSTs learn to observe and examine learning in more detail, to expand how they think about children as mathematical learners including which skills, knowledge, practices and experiences they see as relevant to children's learning, and to help PSTs think about how the knowledge, skills and competencies that children demonstrate in different contexts (school, after school, home, community) might support their school mathematics learning.

Activities:

Activity 1: Mathematics "Getting to Know You" Interview

PSTs conduct an interview with one student in their practicum classroom in an effort to become more familiar with the student's activities and interests, the student's home and community knowledge base and home and community resources. Knowledge gained from this interview can be used to inform the Community Walk activity in the Community Mathematics Exploration Module.

Activity 2: Shadow a Student

PSTs "shadow" a student for a period of time (ideally one full school day) in an effort to identify the child's competencies across contexts. Knowledge gained from this interview can be used to inform the Community Walk activity in the Community Mathematics Exploration Module.

Activity 3: Problem Solving Interviews

PSTs conduct one or more problem solving interviews with one or more students in their practicum classroom. These interviews provide an opportunity to practice eliciting, interpreting, and assessing students' thinking about mathematics, with a particular focus on children's understanding of number concepts. Whole number interview protocols and guidelines were adapted from the work of Tom Carpenter and the Cognitively Guided Instruction (CGI) Group (Carpenter, Fennema, Franke, Levi, & Empson, 1999), as well as work of Susan Empson and colleagues (Empson, Junk, & Turner, 2006). The fraction interview protocol was adapted from work done with Edd Taylor.

Activity 4: Synthesizing and Connecting Across Activities

This assignment is designed to cut across the previous activities in this module. In this write-up, the PST reflects across the multiple interviews and observations that the PST completed with the case study student, and considers how to use this knowledge in mathematics instruction (early thinking about leveraging students' MMKB).

Options and Examples for Sequencing Module Activities throughout a Term

Week	10-Week Course	15-Week Course
2	Introduce Case Study Module; Prepare for *Getting to Know You Interview & Shadowing a Student*	
3	Prepare for *Problem Solving Interview*: Learning about children's strategies for addition and subtraction	Introduce Case Study Module; Prepare for *Getting to Know You Interview & Shadowing a Student*
4	Share experiences in class and submit report for *Getting to Know You Interview & Shadowing a Student;* Continue learning about children's strategies for addition and subtraction.	Share experiences in class and submit report for *Getting to Know You Interview & Shadowing a Student;* Prepare for *Problem Solving Interview*: Learning about children's strategies for addition and subtraction
5	Learning about children's strategies for multiplication and division.	Learning about children's strategies for multiplication and division.
6	Share experiences in class and submit report for *Problem Solving Interviews*	
8	Share and submit Quick Write on *Synthesizing and Connecting Across Activities*	
9	Engage in *Mock Parent/Family Conferences* and submit report.	
10		Share and submit report on *Problem Solving Interview 1* (Addition/Subtraction).
13		Share and submit Quick Write on *Problem Solving Interview 2* (Multiplication/Division).
15		Engage in *Mock Parent/Family Conferences* and submit report on *Synthesizing and Connecting Across Activities*

ACTIVITY 1: GETTING TO KNOW YOU INTERVIEW

Description of Getting to Know You Interview

In this activity, PSTs conduct a single interview with one student in their practicum classroom in an effort to become more familiar with the student's activities and interests, the student's home and community knowledge base, and home and community resources. The goals of this activity are:

- To find out more about the student including the student's interests, activities the student engages in outside of school, and what the student identifies as activities at which the student excels. PSTs might find out, for example, what kind of activities the student engages in with the community with friends and families (i.e., does the student play soccer at a local park, go to a community center, where does the student shop, etc.).
- To identify places, locations, and activities in the community that are familiar to the student, and to find out what the student knows about potential mathematical activity in those settings. These could include locations in the neighborhood immediately surrounding the school (i.e., places that are more or less in walking distance), locations/settings in the neighborhood in which the student lives (if that differs from the community surrounding the school), as well as locations/settings in the broader community with which the student is familiar.
- To find out more about the students' ideas, attitudes and/or dispositions towards mathematics.

Goals for Getting to Know You interview

For the PSTs to become more familiar with the student, his/her interests and activities, his/her home and community knowledge base, and home and community resources. For the PSTs to begin to consider how to use this knowledge in instruction of mathematics.

Lesson Outline for Getting to Know You Interview

LAUNCH.

A brief overview of the assignment is given by the instructor of the methods course. This may include discussions on the following points:

- Explain that PSTs should select a student who is different from them in one or more socio-cultural ways (e.g.., gender, race, socio-economic status, language, etc.). Explain that research indicates that teachers most easily form bonds with students who are like them (Spindler & Spindler, 1982). This interview encourages PSTs to engage and connect with a child with whom they have socio-cultural differences. Explain that this student should be one who struggles at least somewhat with mathematics as this will support the PST in developing strategies for working with students who have been positioned in the classroom as less capable in mathematics.
- How this interview should be a conversation with the child, and that the goal is to learn more about the child, which means probing for examples, asking follow up questions, listening carefully to the child's responses to determine points of further discussion, and so forth.
- How to talk with children about sensitive topics (for example, children may not want to talk about immigration status, if family is undocumented, etc., and/or may not want to

provide detailed information about family or home activities. PSTs should be sensitive to child's desire for privacy and not push such topics).
- PSTs should consult their cooperating/mentor teacher when considering selection of the case study student as well as discuss the case study student with the cooperating/mentor teacher as this teacher might have valuable information to share about the student.

EXPLORE.
The exploration portion of the lesson is the interview that happens outside of the methods class and results in either a) a written assignment that is turned in to the methods instructor, or b) an in class quick write about the activity. Both versions of the write-up (out of class or in class quick write) should include the PSTs' responses to the prompts for this activity. See Handout CS 1, "Case Study Student 'Getting to Know You' Interview" assignment, at the end of this section.

SUMMARIZE.
The summarizing part of the lesson consists of a debriefing discussion in class on the day the assignment is submitted as to what PSTs learned. If there were several PSTs whose field experience was in the same school, it might be good to group them together for the discussion. It might also be effective to group PSTs by grade level.

Instructor Reflection Questions on Getting to Know You Interview. Use your reflections to guide how you support PSTs in future activities and discussions.
- What kinds of things do PSTs tend to notice about students and their communities? (In other words, what do they attend to?) What kinds of things are absent in PSTs' comments and/or written reports? (In other words, what are they not attending to?)
- How are PSTs making sense of what they learn about students and their communities? How are they positioning students' out of school knowledge and experiences? What meaning or value do they seem to assign to mathematical activity in students' homes or communities? (In other words, what kinds of awareness do you notice?)

Ideas from TEACH Math
- We have found that PSTs should select their case student with differences that extend beyond gender. If their only difference is gender, they do not have as substantive of opportunity to consider students' diverse perspectives and experiences.

Additional Resources
Note: The following articles and book chapter are helpful for PSTs to read prior to conducting their getting to know you interview. They support PSTs in gaining a sense of what can be learned about children.

Bartell, T.G. & Meyer, M. R. (2008). Addressing the equity principle in the mathematics classroom. *Mathematics Teacher, 101*(8), 604-608.

Jackson, K. (2009). The social construction of youth in mathematics: The case of a fifth grade classroom. In D. Martin (Ed) *Mathematics teaching, learning, and liberation in the lives of Black children* (pp. 175-199). New York, NY: Routledge.

Kersaint, G. & Chappell, M.F. (2001). Capturing students' interests: A quest to discover mathematics potential. *Teaching Children Mathematics, 7*(9), 512-517.

Martin, D. (2009) Does race matter? *Teaching Children Mathematics 16*(3), 134-139.
Strutchens, M.E. (1999). Data collection: Getting to know your students' attitudes. *Mathematics Teaching in the Middle School, 4*(6), 382-384.

Handout CS 1
Assignment
Case Study Student "Getting to Know You" Interview

Overview of Assignment
There are several purposes of this interview. The **first** is to find out more about the student including student interests, activities the student engages in outside of school, and what the student identifies as activities for which the student excels. You might find out, for example, what kind of activities the student engages in in the community with friends and families (i.e., does the student play soccer at a local park, does the student go to a community center, where does the student shop, etc.). The **second** is to identify places, locations, and activities in the community that are familiar to children, and to find out what the student knows about potential mathematical activity in those settings. These could include locations in the neighborhood immediately surrounding the school (i.e., places that are more or less in walking distance), locations/settings in the neighborhood in which the student lives (if that differs from the community surrounding the school), as well as locations/settings in the broader community with which the student is familiar. A **third** purpose is to find out more about the students' ideas, attitudes and/or dispositions towards mathematics. The interview should last approximately 20-30 minutes.

You will be using the information you gain from this interview when you develop specific contexts for mathematical problems you will present to the student during the Problem Solving Interview. You will also use the information you gain to develop a plan for the Community Mathematics Exploration Assignment you will do later in the semester.

Conducting the interview
Select one student from your practicum classroom who is different from you in one or more socio-cultural ways (i.e., race, socio-economic status, home language; do not select ONLY on the basis of difference in gender) AND who seems to you to struggle at least somewhat with mathematics. Conduct an interview with this student as described above. Below are some interview prompts to assist you.

Suggested interview questions
Use the list of questions below to guide your interview. You do not need to ask all these questions, and you should feel free to adapt the questions as needed. Just keep your goal in mind: to learn more about the student's activities and interests, ideas and dispositions towards mathematics, as well as the community locations and activities that are familiar to students and their families.

<u>Questions about Student's Competencies, Interests, and Activities:</u>

- What things are you really good at? Out of school? In school?
- What do you like to do afterschool? What kinds of things do you like to do at home?
- What are your favorite things to do in school? Favorite subjects?
- What do you like to do at recess?

Questions about Students Home and Community Knowledge Bases and Resources:

- If I was going to walk from the school to your house, what are some things/places that I would see? (This gives you insights into the places that they pay attention to, what they are familiar with, what they notice, etc.)
- What are some places, besides your house, that you like to go after school?
- Where do you like to go with family/friends? What are some places in the community that you like to go to with your family? What do you do there? For example, where do you like to go on the weekends with your family? (This can include places such as grocery or other shopping.)
- What kinds of things do you do with family/friends at home - both regular routines (cooking) and things you enjoy (games)?
- What are some places close to our school that you have been to? What do you do at those places? (Can you think of any other places close to our school? Have you ever been to those places? What do people do there? What do you do there?)
- Can you think of any places in your community where people do math or use math? What about your family members, where do they use math? Where do they do math?

Questions about Students' Ideas/Dispositions Related to Math:

- What are some things in math that you are really good at? What is something in math that you are not as good at? How do you know?
- What are some things in math that you really like? What about math do you not like, if anything?
- Do you think it is important to learn math? Why do you think it important to learn math?
- How would you describe what math is? (e.g. describe to younger sibling, or contrast it with another school subject)
- Who do you know that is good in math (in general, in class)? How do you know? What makes them good?
- Why do you think it is hard for some people to learn math?
- Do you use math outside of school? How?
- Who helps you with math if you get stuck in class? How about home? How do they help you?
- Have you learned math in a different school? Country? How was it similar or different?

In a paper of one to three pages, please respond to EACH of the following questions and sub-questions:

A. **Introduce your student (using first name only or a pseudonym):** Include age, grade, school, gender, race/ethnicity, family background, home language(s), etc. Explain why you selected this student (and compare/contrast the students' background with yours, noting the socio-cultural difference/s between you two). This part is intended to be a brief introduction only. (1 short paragraph)

B. Learning about Student. *What did you learn about your student that you did not know before?* More specifically:

- what did you learn about your student's competencies, interests, and activities,
- the student's ideas/dispositions towards mathematics,
- and the student's family and/or community based knowledge and experiences?

- Where in the student's community might provide a good site for exploring mathematics?

C. Learning about Self. *What did you learn about yourself in this process?*

- Were any of your assumptions about students, their activities/interests, their communities/families, and their ideas/dispositions about math affirmed and/or challenged by participating in this process?

D. Links to course readings. *How do you relate what you learned to one or more of our class readings.*

Activity 2: Shadow a Student Activity

Description of Shadow a Student

In this assignment (Handout CS 2), PSTs are asked to "shadow" a student for a period of time (ideally, one day, but they could break up the time over two days; 6 hours of total shadowing time is preferred, of which a maximum of 2 hours are in math class). The critical aspect is that NOT ALL shadowing occurs during math class and that shadowing does NOT occur while the PST is teaching a lesson or conducting an activity with students. The goal of this activity is for the PSTs to:

- Become more familiar with the student with whom they conducted the "Getting to Know You Interview," both in terms of their mathematics knowledge and in terms of their interests, competencies, culture, and life experiences.
- Consider how to use this knowledge in instruction of mathematics (early thinking about leveraging students' MMKB).

Goals for shadow a student activity
For the PSTs to become more familiar with a particular student in their practicum placement both in terms of their mathematics knowledge and in terms of their interests, culture, and life experiences. For the PSTs to begin to consider how to use this knowledge in instruction of mathematics.

Lesson Outline for Shadowing a Student

LAUNCH:
The instructor provides a brief description of the assignment, sharing the above overview and goals. Then the instructor asks/discusses the following with PSTs:

- What the PST should look for/observe when shadowing a student. Discuss what we mean by competence, and that we are interested in all kinds of competencies, not just "traditional school math" related competencies. In some instances, PSTs have conflated confidence with competence, so it is important to explicitly discuss what we mean by competencies. Emphasize that PSTs should attend to the child's strengths and competencies.
- PSTs should expect to learn things when they shadow a child that they did not learn from their interview – shadowing provides additional insights, allows them to see the child in multiple contexts, in action, all things that are not possible in the getting to know you interview.

EXPLORE:
The Explore part of this activity happens outside of the methods class and results in a written assignment that is turned in to the methods instructor. (Alternatively, PSTs could complete an in class quick write in which they reflect on the shadow a student experience). Either version (out of class write up or in class quick write) should include responses to the essential prompts for this activity.

SUMMARIZE:
The Summary part consists of a debriefing discussion in class on the day the assignment is submitted. If several PSTs completed field experiences in the same school, consider grouping them together for the discussion. Another option is to group PSTs by grade level. Present the following prompts to guide small group discussions, and then follow with whole class sharing from the small groups. In whole class, ask for highlights, common themes, emerging questions, and insights from the small group discussions.

- *What did you learn about your student in this activity?* More specifically, what did you learn about your students' strengths and competencies? Consider the various strengths and competencies that your student exhibited across the various settings in which you conducted your observations.
- *How can you use what you learned about the student to inform your mathematics teaching?* Describe in detail some ways you could use what you learned about this student in your teaching. Describe some problem contexts, or some efforts you'd make, to include and involve this student, and to build upon this students' strengths and competencies in mathematics class.
- *What did you learn about yourself in this process? What did you think of the process itself?* Did you learn anything about yourself in this process? Was it difficult to shadow a student? What was challenging? What went well? Do you think this is something you could continue to do with students in your mathematics class? Did shadowing a student allow you to get to know the student any better? Did shadowing the student seem to affect the student's participation in classes or the student in general?

Instructor Reflections on Shadow a Student
Use your reflections to guide how you support PSTs in future activities and discussions.

- What kinds of things do PSTs tend to notice about students? What kinds of competencies did they notice? What kinds of participation did they notice? What did they notice about students' interactions with their peers? (In other words, what do they attend to?) What kinds of things are absent in PSTs' comments and/or written reports? (In other words, what are they not attending to? How might you support PSTs in expanding what they attend to)?
- How are PSTs making sense of what they learn about students? How are they positioning students' competencies in terms of their relevance to mathematics learning? What meaning or value do they seem to assign to the ways in which students participate, the nature of their social interactions, the range of skills and competencies they demonstrate (including those that may not typically be seen as relevant to mathematics? (In other words, what kinds of awareness do you notice? How might you support PSTs in further developing these awarenesses?)

Ideas from TEACH Math

- We have often combined the "Getting to Know You Interview" with the "Shadow a Student" activities as two ways to learn about case study students within one assignment, and then PSTs can draw from both experiences in writing and discussing their ideas for instruction and their reflections on the student.

Additional Resources for Instructors for Shadow a Student

Note: This article provides background for the MTE that may be useful to read before using this activity the PSTs.

Soto-Hinman, I. (2010). ELL shadowing: Strengthening pedagogy and practice with pre-service and in-service teachers. *Research in Higher Education Journal 8,* 1-11.

Handout CS 2
Assignment
Shadowing a Student

Quality mathematics teaching requires that you know your students well, both in terms of their mathematics knowledge and in terms of their interests, culture, and life experiences. In this assignment, you are asked to "shadow" a student for a period of time (ideally, one day, but you could break up the time over two days).

Choose a student that you would like to get to know a bit better through observation and informal discussion. Ask that student if you can "shadow" them for a day (or parts of two days). Tell them that you would like them to show you who their friends are, what they are good at doing, and other things important to them in a regular school day. Ask if you can watch them in the hallways between classes, if you can watch them at lunch interacting with friends, if you can observe their participation in other classes, in before or after school activities, and if you can ask them a few questions about themselves.

Pay attention to some of the following details. This is by no means an exhaustive list.

- When does the student participate in the class, and when does the student refrain from participation? Is the student's participation different in different classes?
- In what ways does the student participate in his/her various classes?
- Who does the student seem to know or be friends with in his/her various classes? Does the student also seem to associate with these friends in the hallways or at lunch? If possible, talk to the student's friends. What do they like about him/her?
- Do you see the student in the halls or during breaks or recess? If so, how does she/he act in the halls? Who is he/she with? How does he/she respond when she/he sees you? What sorts of topics are students discussing in the halls/at recess (that's right, listen to the conversations, which might be uncomfortable or alarming to you!).
- In listening to the student talk about himself/herself and the friends talk to or about the student, does it seem like the student is "known for" anything in particular (e.g., good athlete, good at math, good reader, artist, "popular" with other kids, in a certain "group", etc)? (Observing for the student's perceived "identity.")
- What is the student good at doing? In what ways does the student demonstrate his/her competence across these various settings?
- What does the student report he/she is good at doing?
- What does schooling look like from the student's perspective?
- In what ways is the student showing *competence* (i.e., skill, ability, knowledge, creativity, problem-solving, reasoning) in various contexts? Pay particular attention to ways you see the student showing his/her competence that might not be captured by a teacher or by a task.

Write Up Prompts:

D. **Introduce your student:** Include age, grade, school, gender, race/ethnicity, family background, home language(s), etc. Explain why you selected this student (and compare/contrast the students' background with yours). This part is intended to be a brief introduction only. (1 short paragraph) [[NOTE: may omit this section as long as this information is incorporated in a later write up]]

E. **Methods of Shadowing.** *Describe the shadowing experience*. Specify the duration of observations, contexts in which the student was observed, and nature of interactions with the student. How did you first approach the student to ask him/her to shadow them? Did you talk with the student in the various settings? Did you meet the student's friends or family? Where did you observe the student? (1 short paragraph)

F. Learning about Student. *What did you learn about your student that you did not know before?* Pay specific attention to ways the student discusses or reveals his/her strengths and competence across settings. What did you learn about your student's interests and activities, his/her ideas/dispositions towards mathematics, and/or his/her family and/or community based knowledge and experiences? If you could observe the student again, what would you want to be sure to notice?

G. Learning about Self. *What did you learn about yourself in this process?* Were any of your assumptions and/or biases about students, their activities/interests, their communities/families, and their ideas/dispositions about math affirmed and/or challenged by participating in this process?

H. Connection to mathematics teaching. *How can you use what you learned about the student to inform your mathematics teaching?* Describe some problem contexts, or some efforts you could make, to include and involve this student in mathematics class, and to feature this student's experiences, interests, knowledge and strengths. Use specific examples.

ACTIVITY 3: PROBLEM SOLVING INTERVIEW/S

Description of Problem Solving Interview/s

PSTs conduct a single problem solving interview or a series of interviews with one or more students in their practicum classroom. The grade level of the child is not important, but one of the students should be the SAME student that participated in Activity 1 ("Getting to Know You" Interview) and/or Activity 2 (Shadow a Student) of this module. These interviews provide an opportunity to practice eliciting, interpreting, and assessing student thinking about mathematics, with a particular focus on children's understanding of number concepts. The whole number interview protocols and guidelines were adapted from the work of Tom Carpenter and the Cognitively Guided Instruction (CGI) Group (Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999)), as well as other work of Susan Empson (Empson, Junk, & Turner, 2006). The fraction interview protocol was adapted from the work of Edd Taylor.

Each problem solving interview should take about 20-25 minutes, and should be conducted in a relatively quiet place. PSTs should take detailed notes during the interview and collect all student work. Many elementary schools do not permit audio recordings of students but if it is allowed, these recordings can be helpful.

At the end of this section you will find several sample interview protocols that PSTs can use to guide the interviews. The protocols are designed to be adaptable to children at various grade levels and/or according to prior mathematical experiences. PSTs should also be encouraged to adapt the suggested problems (context, numbers, wording, language, etc.) to meet the needs of their student.

Goals for Problem Solving Interviews

For the PSTs to learn more about an individual child's mathematical thinking, including their strategies for solving problems and their understanding of particular content areas. The activity also provides PSTs with an opportunity to practice eliciting, clarifying and extending students' thinking about mathematics.

Lesson Outline for Problem Solving Interview/s

LAUNCH.

The launch for this activity includes four components: a) reviewing the interview protocols and procedures for conducting the interview (Handout CS 3), b) reviewing possible interview scenarios, c) viewing and discussing video clips of sample one-on-one interviews, and d) discussing relevant readings.

- *Reviewing problems and problem sequences.* Ask PSTs to spend time in or out of class reviewing each of the problems in the interview protocol (see Handout CS 4, "Problem Sequence for Addition/Subtraction and Multiplication/Division Problems"; Handout CS 5, "Addition/Subtraction and Multiplication/Division Problems"; Handout CS 6, "Fraction Interview Problems"; Handout CS 7, "Bare Number Fraction Problems"; Handout CS 8, "Base 10 Concepts 1"; and Handout CS 9, "Base 10 Concepts 2"), understanding the problem sequence, selecting number choices that they think would be appropriate for the child they will interview, and changing problem contexts so that they might be more be appropriate for the child they will interview. Next, ask PSTs to predict

2-3 different strategies that their student might use to solve each of the problems. PSTs might be placed in groups according to the grade level of the child for this work. NOTE: There are a range of possible interview combinations. For example, one could (a) do one K-2 interview on addition/subtraction and one 3-5 interview on multiplication and division, (b) do one interview on the four basic operations and a second interview on fractions, or place value, or (c) do a single problem solving interview that incorporates all four operations.

- *Reviewing possible interview scenarios.* Discuss scenarios that might arise during the interview, and how PSTs might respond. See Handout CS 10, "Interview Scenarios" (adapted from Empson and colleagues (2006)) at the end of this section. PSTs might first discuss all scenarios in small groups, and then as a whole group, discuss 3-4 scenarios that PSTs have questions about, and/or 3-4 that the instructor feels are particularly important to address.

- *Viewing videos.* Show several video clips of sample problem solving interviews. While videos from the CGI materials (Carpenter, et. al, 1999) are useful in terms of demonstrating particular strategies that students use, there are other videos that model how to conduct an interview and/or how instructors respond to various scenarios that might arise. Productive clips are ones that
    - Show how an interviewer STARTS an interview (how the choice of materials are explained, how he/she introduces self to child and talks to child about the purpose of the interview, etc.)
    - Show students who guess, get the problem wrong, and/or are stuck at some point in the solution, and then show how the interviewer responds.

  Below is a list of sample clips that might be used.
    - Clips from Randy Philipp's Interview with Nicole (available on the IMAP video clip collection); Clips 1, 21, 22 are good examples.
    - Finally, video clips from the CGI and/or IMAP DVDs might be used to provide examples of particular strategies. For the most part, these clips are not useful in terms of modeling interviewing techniques, but are very useful in terms of exemplifying different strategies (direct modeling, counting, various alternate algorithms, etc.)

- *Discussion of relevant readings.* To help PSTs prepare for conducting the interview, they might read and discuss an article such as Ginsburg (1997) "Assessing Thinking: What does the child know?" PSTs should focus on Ginsburg's suggestions for what to do and not do during the interview, and on Ginsburg's emphasis on always looking for the sense of students' comments and strategies (even if the logic is not immediately obvious.) Other relevant readings include articles/text chapters that describe the basic addition, subtraction, multiplication and division problem types, and students' strategies for solving those problems. For example, PSTs might read and discuss relevant chapters from the *Children's Mathematics* (Carpenter et al., 1999) before conducting the interview.

EXPLORE.
The problem solving interviews with the elementary school students are conducted in the elementary school. See Handouts CS 5 – CS 9.

SUMMARIZE.
For the class session after PSTs have conducted the interview, ask PSTs to bring their notes from the interview to class, as well as copies of the work that the student produced during the interview (pictures, paper and pencil algorithms, etc). PSTs meet in small grade-level groups to discuss a) the strategies that students used (with attention to the details of what students did to solve a particular problem), and b) what PSTs learned about their student's understanding based on their interactions during the interview (with an emphasis on making specific claims about a child's understanding and supporting those claims with evidence). These in-class small group discussions function to prepare PSTs for the interview write-ups they will produce.

Small and/or whole group discussion might also focus on identifying the particular strategies that students used (direct modeling, counting, etc.) and/or on identifying the particular teacher moves that PSTs used during the interview to support, clarify and/or extend the student's thinking. (Reading the Jacobs & Ambrose (2008) article that outlines supporting and extending moves, and gives examples of how teachers use these moves in one-on-one interactions with students would help prepare PSTs for this discussion.)

In a written report following each enactment of a problem solving interview, PSTs summarize what they learned about the student's mathematical thinking and how they might use what they learned to guide their instruction. This can be structured as an out of class assignment that will be submitted to the instructor, or as an in-class quick write and/or discussion.

Instructor Reflections on Problem Solving Interview/s. Use your reflections to guide how you support PSTs in future activities and discussions.

- What kinds of things do PSTs tend to notice about students' strategies and mathematical thinking? How do they attend to/respond to strategies and/or algorithms that may be unfamiliar and/or unexpected? What do they notice about the nuances of students' strategies? (In other words, what do they attend to? And in what detail do they attend to students' mathematical thinking?) What kinds of things are absent in PSTs comments and/or written reports? (In other words, what are they not attending to or what details are they missing?)

- How are PSTs making sense of what they learn about students' strategies and understanding? What kind of claims are they able to make about students' understanding, and how do they use evidence (or not) to support those claims? To what extent do they make claims about what students DO understand versus, or in addition to, claims about what a student does not understand? (In other words, what kinds of awareness do you notice?)

Additional Resources for Problem Solving Interviews

Note: With the exception of the article by Perkins & Flores, the following articles and book chapter provide additional background on children's mathematical thinking. These would be useful if you are interested in further work that builds on the ideas of Carpenter and colleagues

(1999). The Perkins article focuses specifically on mathematical notation from around the world. PSTs are often surprised to learn that standard US notation is not universal; this article provides support for making sense of notation that children might bring to the classroom from other learning experiences.

Ellemor-Collins, D.L. & Wright, R.J. (2008). Assessing student thinking about arithmetic: Videotaped interviews. *Teaching Children Mathematics, 15*(2), 106-111.

Jacobs, V. & Kusiak, J. (2006, May) Exploring children's use of tools during problem solving. *Teaching Children Mathematics, 12*(9), 470-477.

Lo Cicero, A., Fuson, K & Allexsaht-Snider, M. (2001). Mathematizing children's stories, helping children solve word problems, and supporting parent involvement. In *Changing the faces of mathematics: Perspectives on Latinos* (pp. 59-70). Reston, VA: National Council of Teachers of Mathematics.

Perkins, I & Flores, A. (2002, February). Mathematical notations and procedures of recent immigrant students. *Mathematics Teaching in the Middle School,* 7(6), 346-351.

Watanabe, T. (1996). Ben's understanding of one-half. *Teaching Children Mathematics, 2*(8), 460-464.

HANDOUT CS 3
PROBLEM SOLVING INTERVIEW GUIDELINES

This student interview assignment is designed to give you an opportunity to focus on individual children's mathematical thinking.

How do I set up the interviews?

Conduct the interview with your case study student, or with your case study student and another child if you are interviewing two children. It does not matter whether the children solve the problems correctly or how many problems the children solve correctly. You will learn something from all children. Identify a quiet location to conduct the interviews.

How do I conduct the interviews?

Have materials available for the child to work with (paper, pencil/pen/marker, and manipulatives such as base 10 blocks and multilinks).

Work with the child/children individually. Begin the interview by informing the child that you will be asking him/her to complete a few mathematics problems. Let the child know that you are just as interested in the strategies used to solve the problem as you are in the answer. Tell the child that s/he can solve the problem in any way that makes sense to him/her and using any of the materials you have provided.

Read each of the problems to the child – one at a time – and provide him/her with sufficient time to complete each part of the problem. You can also provide each child with a written copy of each problem, but you MUST read the problems aloud in this case as well. (Only give the child one written problem at a time, not the whole interview problem set.)

After the child answers each problem, you should ask a variety of questions that will help you to better understand the child's thinking and to assess his/her mathematical understanding. For example, if the child is not forthcoming with a response or says "I just knew it," you might respond with "What did you think about first?" or "If you were helping a friend, how would you explain what you did?"

This assignment will provide you with problems to pose, but you may make changes to any of these problems if necessary. You may also want to encourage a child to solve a problem in more than one way. If you make changes to the problems provided, be sure to indicate why you made those changes in your write-up. Be careful not to change the underlying mathematics.

Make sure the child leaves feeling successful. If you believe the child is too frustrated, you may end work on that problem or the entire interview early. Before the child leaves, you may want to pose a final (easy) problem that you are sure the child can solve or have him/her make up a problem to solve. If you choose not to pose all of the interview problems, you should indicate your reasoning in your write-up.

Interviewing Tips

Be Curious! The point of the interview is to discover how the child thinks – *not* to guide the child to the correct answer. (For the purpose of this assignment, it does not matter if the child solves everything incorrectly – you will still learn something about her/his thinking.)

This is not an opportunity to teach. Your job during the interview is to ask the students to work on the mathematical task you present and ask questions only to understand their thinking. Do not help the students solve the problems. If you think you might be tempted to teach, give the student the mathematical task, leave the student alone, and only ask questions when he or she is done with the mathematical task.

Be careful to respond similarly to correct and incorrect answers. Be curious about all responses.

Your primary role is to listen. Make sure you allow enough "wait time" – waiting can often be hard (!) but children need time to think before answering.

What if a child cannot solve a problem?

You can try to clarify what the problem is asking.

You can try asking about any partial strategies the child has attempted. Sometimes talking about a problem or strategy can help a child get unstuck.

You can try asking the problem again, keeping the structure of the problem the same, but making the context more familiar (e.g., putting the child's name or items the child knows well in the problem.)

You can try changing the problem to a similar problem with smaller numbers.

You can move on to the next problem and write about what you learned from this experience.

What problems should I use?

See the attached pages for problems to use with your students.

Additional Tips on Supporting, Extending, and Clarifying Students' Thinking

Note: This framework developed by Vicki Jacobs and Rebecca Ambrose. This framework connects to the Jacobs and Ambrose article (2008) from Teaching Children Mathematics.

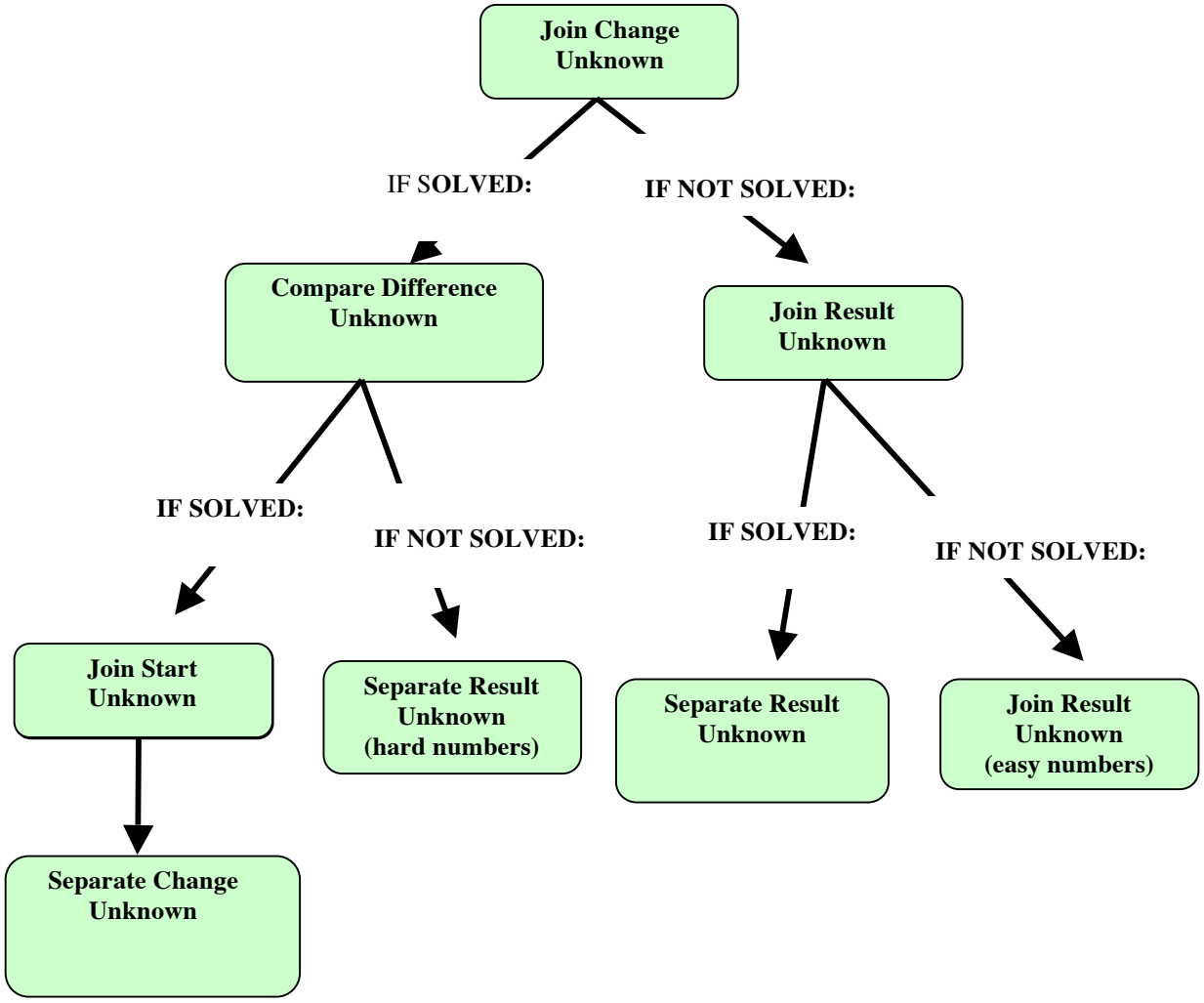
How to support a child while s/he is solving a problem	Make sure the child understands the problem	• Ask the child to explain what s/he knows about the problem • Explain vocabulary • Rephrase or embellish the problem
	Change the problem when necessary	• Make the numbers easier • Make the problem context more familiar • Make the problem structure easier • Abandon the problem
	Explore what the child has already done	• Ask the child to explain what s/he did • Ask specific questions about how what the child did relates to the problem

	Remind the child that other strategies are possible	• Ask the child to try a different strategy • Ask the child to try a different tool • Remind the child of relevant strategies s/he has used before
How to clarify a teacher's understanding of a child's strategy		• Ask the child to explain (or re-enact) how s/he solved the problem • Ask specific questions to clarify parts of the strategy
How to extend a child's thinking after solving a problem	Promote reflection on the strategy just completed	• Ask the child to re-explain the strategy to reinforce his/her understanding • Ask specific questions about how parts of the child's strategy relate to the problem
	Encourage the child to explore multiple strategies and their connections	• Ask if the child could count differently (or faster) • Ask the child to try a second strategy • Ask the child to try a more difficult or efficient strategy • Ask the child to compare strategies
	Connect the child's thinking to symbolic notation	• Ask the child to write a number sentence that represents problem • Ask the child to record his/her strategy
	Generate follow-up problems linked to the problem just completed	• Ask the child to write a similar problem • Ask the child to solve a similar problem, perhaps with more difficult numbers • Ask the child to solve a series of problems and explore their connections

Handout CS 4
Problem Sequence for Addition/Subtraction and Multiplication/Division Problems

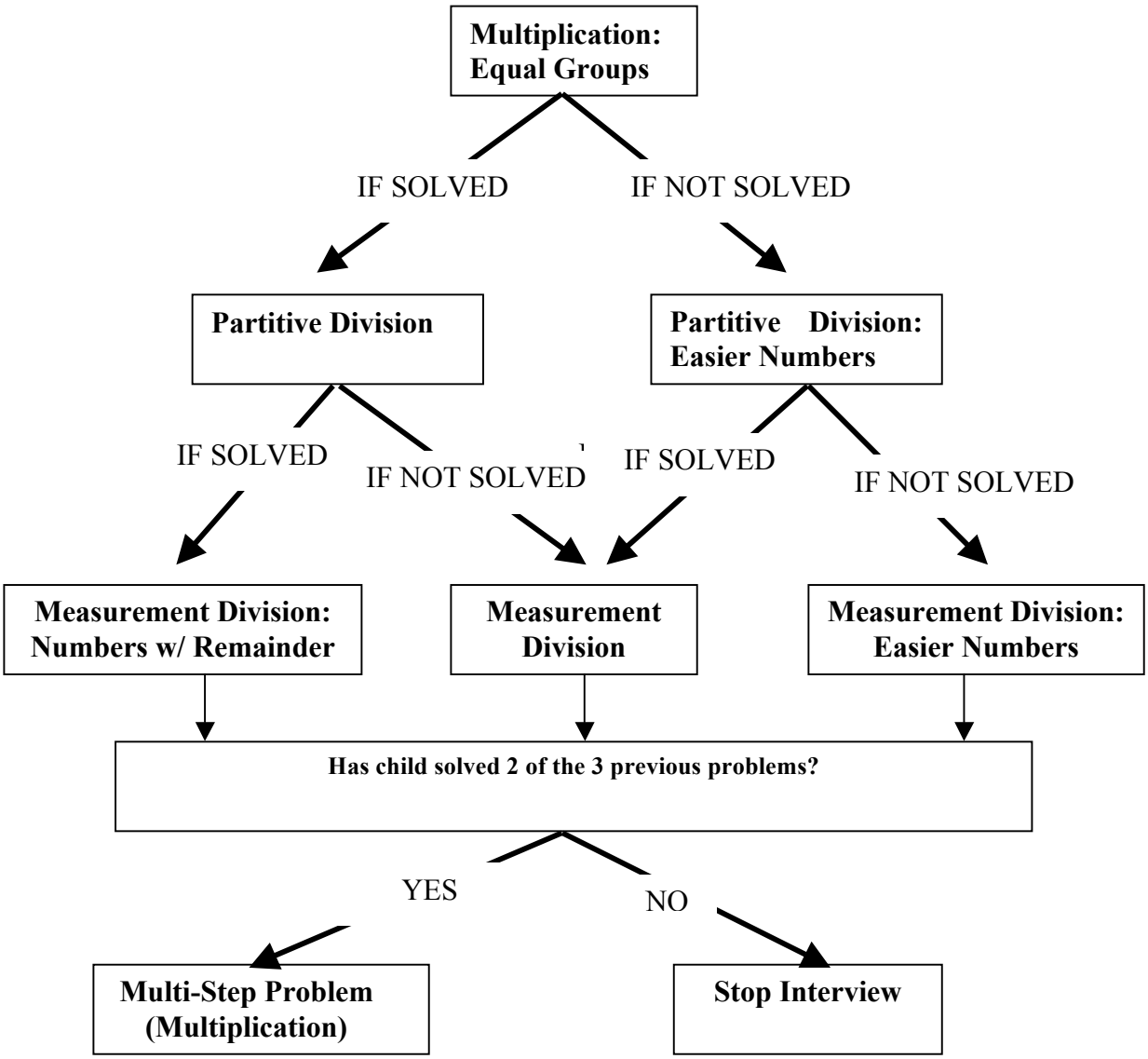
Part 1: Addition/Subtraction

Problem Sequence: Here is a decision tree for the Addition/Subtraction Problems, to show which problems to do when. Each child solves at least 3 problems in this portion of the assessment.



Part 2: Multiplication and Division

Problem Sequence: Here is a decision tree for the Multiplication/Division Problems to show which problems to do when. Children will solve either 3 or 4 problems in this portion of the assessment.



Handout CS 5
Problem Solving Interview
Addition/Subtraction and Multiplication/Division Problems

Your Name: _____ Child's First Name:_____

Date:_____ Child's Grade:_____

Part 1: Addition/Subtraction

A. JOIN CHANGE UNKNONWN
[IF solved, go to problem C. If not solved go to problem B]

Eric is saving money to buy a present for his brother. The present costs 7 dollars. He has 4 dollars so far. How many more dollars does Eric need to buy the present?

Alternate numbers: Easier: (5, 4) Harder: (17, 8; 28, 17) Even Harder: (124, 89)
Older Student: (357, 249; $341.00, $263.45)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

B. JOIN RESULT UNKNONWN
[IF solved, go to problem D, If not solved, try again with easier numbers, then try problem D with easier numbers]

Maya has 2 jellybeans. Her brother gives her 4 more jellybeans. How many jellybeans does Maya have now?

Alternate numbers: Easier: (1, 3) Harder: (12, 8; 36, 18) Even Harder: (198, 79)
Older Student: (763, 399)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

C. COMPARE DIFFERENCE UNKNOWN
[IF solved, go to problem E and then F. If not solved go to problem D, try with harder numbers.]

Sarita has 9 toy cars. Her brother George has 6 toy cars. How many more toy cars does Sarita have than George?

Alternate numbers: Easier: (5, 3) Harder: (14, 6) Even Harder: (42, 20)
Older Student: (182, 96; 702, 468)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

D. SEPARATE RESULT UNKNOWN
[IF solved, end interview, or try again with harder numbers. If not solved try with easier numbers, and then end interview.]

Jason has 8 pennies. He loses 3 of them. How many pennies does Jason have now?

Alternate numbers: Easier: (5, 2) Harder: (13, 4) Even Harder (54, 28)
Older Student: (301, 157; 1255, 459)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

E. JOIN START UNKNOWN
[IF solved, go to problem F. If not solved try with easier numbers, and then end interview.]

Marcos had some stickers. Then, for his birthday, his friends gave him 3 more stickers. Now he has 8 stickers altogether. How many stickers did Marcos have before his friends gave him some for his birthday?

Alternate numbers: Easier: (2, 4) Harder: (9, 13) Harder: (12, 25)
Older Student: (39, 105; 121, 248)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

F. ADDITIONAL PROBLEM: SEPARATE CHANGE UNKNOWN
[IF solved, end interview, or try again with harder numbers. If not solved, try with easier numbers, and then end interview.]

There were 8 kids playing soccer. Then some kids left to go home. Now there are only 3 kids playing soccer. How many kids left to go home?

Alternate numbers: Easier: (2, 4) Harder: (9, 13) Even Harder: (44, 35)
Older Student: (572, 385)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

Part 2: Multiplication and Division

NOTE: Problem Difficulty

Each problem is written with numbers that are at a middle level of difficulty (for 1st or 2nd graders). Also included are easier and harder options. The harder level numbers are appropriate for 3rd - 6th graders. However, this is not an absolute division. If a child at any grade level is having trouble, you could substitute easier numbers. If a child at any grade level is succeeding easily, you could try some of the problems with harder numbers.

A. MULTIPLICATION: Equal Groups

Jesse has 4 pockets. He puts 3 pennies in each pocket. How many pennies does Jesse have?

Alternate numbers: Easier: (2, 3) Hard: (9, 4) Harder: (15, 8) Hardest: (12, 28; 32, 44)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

B. PARTITIVE DIVISION

There are 20 marbles that 4 friends want to share evenly. Each friend wants to have the same number. How many marbles can each friend have?

Alternate numbers: Easier: (8, 4) or (6, 2) or (4, 2) Hard: (30, 6) Harder: (84, 7 OR 120, 8) Hardest: (384, 24)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

C. MEASUREMENT DIVISION

Becky has 15 cookies. She wants to put the cookies onto plates, with 5 cookies on each plate. How many plates does she need?

Easier: (10, 2) or (6, 2)

Hard. NO Remainder: (132, 12)

Easier with Remainder: (7,2)
Medium With Remainder: (23, 10)
Hard With Remainder: (110, 12)
Even Harder with Remainder: (292, 16)

Numbers Used:_____ Child's Answer:_____

Description of Strategy:

D. MULTI-STEP MULTIPLICATION PROBLEM

Lily has 3 boxes of chocolates. Each box has 4 pieces of chocolate. If Lily gives 5 pieces of chocolate to her sister, how many pieces of chocolate will she have then?

Easier: (4, 2, 3) Hard: (9, 8, 14) Even Harder: (16, 8, 12 OR 32, 14, 25)

Numbers Used: _____ Child's Answer:_____
Description of Strategy:

Additional Problems for Whole Number Problem Solving Interviews

Addition/Subtraction

Paco had 13 cookies. He ate 6 of them. How many cookies does Paco have left?
(*alternate numbers:* 43, 16)

Lisa had 3 stickers in her sticker collection. She got 4 more stickers for her birthday. How many stickers does she have in her collection now?

Tom has 10 teddy bears. His friend Juan gave him 2 more teddy bears. How many teddy bears does Tom have now?

There are 4 boys and 4 girls on the playground. How many children are on the playground?

Hannah has 12 balloons. Jacob has 7 balloons. How many more balloons does Hannah have than Jacob?
(*alternate numbers:* 42, 29)

Carla has 7 dollars. How many more dollars does she have to earn so that she will have 11 dollars to buy a puppy?
(*alternate numbers:* 27, 48)

You had 40 buttons in your collection but then you lost 17 of them. How many buttons do you have in your collection now?
(*alternate numbers:* 400, 294)

There are 23 books on the shelf. Marta put 38 more books on the shelf. How many books are on the shelf now?(*alternate numbers:* 151, 67)

Matt had 18 pennies in his bank. For his birthday, he got 25 more pennies. How many pennies does Matt have now?
(*alternate numbers:* (10,11) (34,20) (32,63) (80,40) (99,26) (198,282))

Multiplication/Division

Mr. Gomez had 20 cupcakes. He put the cupcakes into 4 boxes so that there was the same number of cupcakes in each box. How many cupcakes did Mr. Gomez put in each box?
(*alternate numbers:* 42, 3)

Robin has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Robin have altogether?
(*alternate numbers:* 6, 12)

Tad had 15 guppies. He put 3 guppies in each jar. How many jars did Tad put guppies in?
(*alternate numbers:* 54, 3)

If 10 donuts fit on a plate, how many plates would we need for 87 donuts?

(*alternate numbers:* 10, 287)

Arthur has 4 packets of seeds with 11 seeds in each packet. How many seeds does he have altogether? (*alternate numbers:* 15, 21)

19 children are taking a mini-bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit 3 to a seat and how many can sit 2 to a seat?

Base 10

We have 4 boxes of chalk with 10 pieces of chalk in each box. We also have 6 loose pieces of chalk. How many pieces of chalk do we have?
(*alternate numbers:* 24 boxes, 10 crayons in each box, 6 extra crayons)

Handout CS 6
Contextual Fraction Interview Problems

- For young children ages 5-7 (Grades K-2)
  - Use Task 2 from the Watanabe (1996) article "Choosing the Largest 'Cookie' "
    - **Note: you need to make the cookie pieces out of card stock or construction paper.**

- For older children ages 7-9 (Grades 2-4)
  - Choose between
    - Task 1 from the Watanabe (1996) article "Half-Colored Shapes"

    - This task: There are 4 children playing together. They have 7 brownies to share for a snack. How much will each child get if they each get an equal share?
      - **Note: Provide file cards or other small paper rectangles to be used as brownies**

- For oldest children ages 9-11 (Grades 4-6)
  - Choose among
    - One of the above tasks may be appropriate if the child is learner who is struggling with concepts in mathematics.

    - This task: You have 1 3/4 yards of fabric to make puppets. It takes 1/2 yard for each puppet. How many puppets can you make out of your fabric?
      - **Note: Provide a strip of paper (old adding machine tape, for example) to simulate the fabric if the child chooses to use a manipulative.**

    - This task: Jose had 4 whole brownies and 1/2 of another piece. He decided to give some friends 3/4 of a brownie for a treat. To how many friends did Jose give this treat?
      - **Note: Provide file cards or other small paper rectangles to be used as brownies if the child chooses.**

Watanabe, T. (1996). Ben's understanding of one-half. *Teaching Children Mathematics, 2*(8), 460-464.

Handout CS 7
Bare Number Fraction Interview Problems

Put these fractions on the number line below

2/3 2/12 4/4 3/12 5/6 1/6 1/2 3/4

Find the sum of 1/6 + 1/2. (alternate 3/4 + 1/2)

Prove that 1/2 + 1/4 = 9/12

Handout CS 8
Problem Solving Interview: Base Ten Concepts 1

For each question, 1) *Read the problem to the child.* ***Re-read as many times as the child needs. Read the entire problem each time.*** *Do not read it in parts. You may have a written version available for the child, but this is optional.* 2) *Allow the child to use manipulatives, their fingers, paper and pencil, a mental strategy, or anything else they would like to solve the problem.* 3) *Ask the child, "How did you figure that out?" when he or she is finished solving the problem. Probe the child's thinking until you understand exactly how he or she solved the problem.* ***(Respond to incorrect answers in the same way.)***	**Materials** **Unifix cubes (about 4 stacks of 10 and 10 loose ones), and/or base ten blocks, paper and pencil, base ten pictures.**

Goal

Your goal in this assessment is to find out what the student understands, without your assistance. If a child is having difficulty with a problem, you may change the numbers or move on to a new problem type. There is no need to show students how to solve a problem. If you feel the child needs some support, you can just say something like, " Really good try ____. We are going to work on this some more this year." Or " This is a hard problem, it is okay if you do not finish it right now."

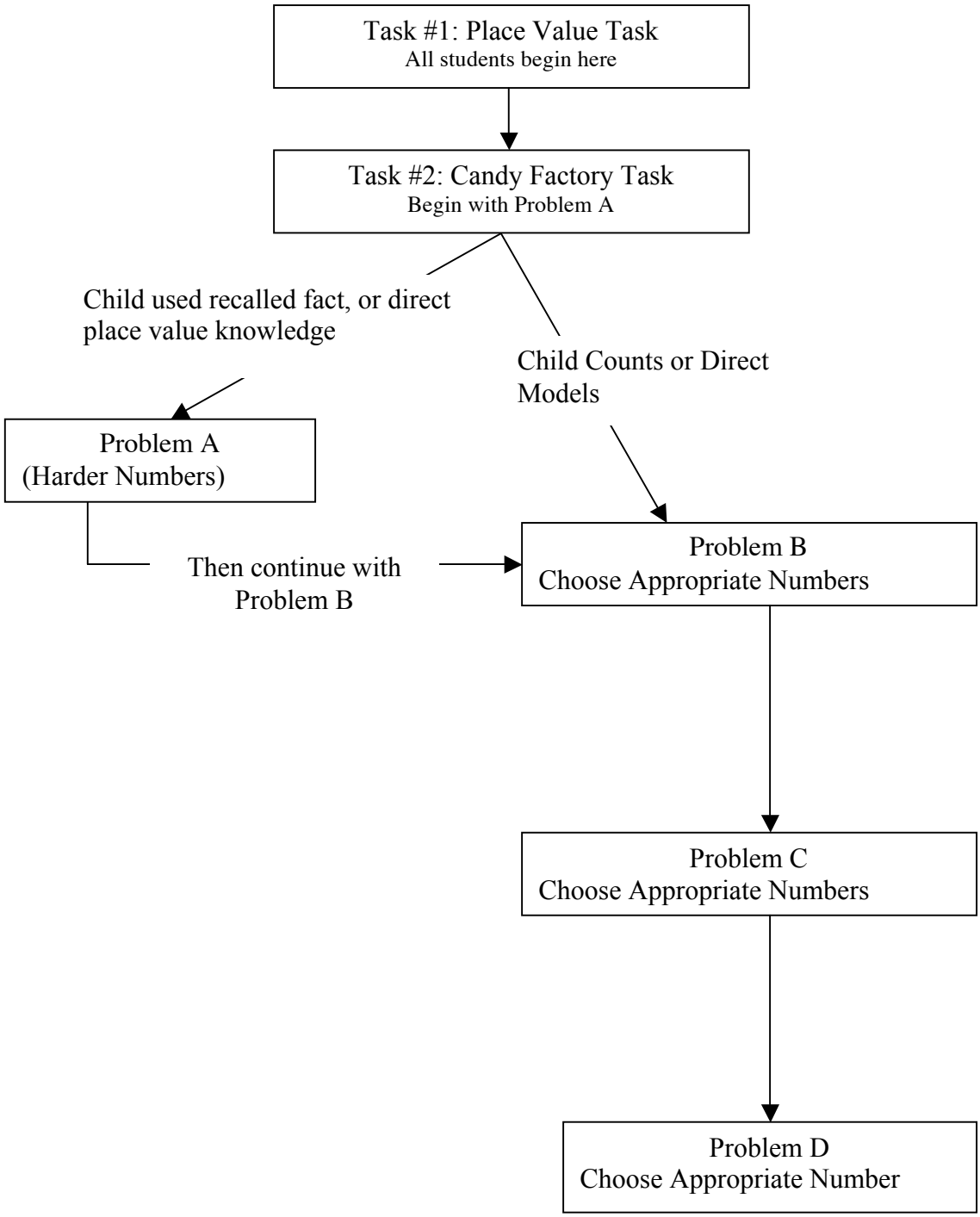
Problems

The Base Ten Concepts 1 interview includes 2 kinds of tasks. All children will begin with Task #1, which is a brief place value task. For Task #2, we made a decision tree to show which problems to do when, according to the strategies the child uses. Children will solve between 3 and 4 problems in this assessment. If a child at any grade level is having trouble, you could substitute easier numbers. If a child at any grade level is succeeding easily, you could try some of the problems with harder numbers.

Administration of the Assessment:

This assessment is designed for teachers to give one on one with students. Each assessment should take about 15 minutes on average since you will only give three to four problems for each assessment. This can be done in a corner of the classroom as other students are working independently (much as you might do with a reading record), or you can find another space such as a hallway or other area.

Base Ten Concepts Assessment: Decision Tree for Interview



Task #1: Place Value Task

Put a pile of individual cubes or counters (more than 32) on the table.

Show the number 32 written on a card.
Ask child to read the number.
Ask the child to show the number with cubes.

Then point to the 3 on the cards and ask "What does this part mean? Could you show me with the cubes what this part means?"

Similarly, point the 2 and ask "What does this part mean? Could you show me with the cubes what this part means?"

_____ Demonstrates place value understanding (the 3 represents 30 cubes, or 3 tens)

Describe child's thinking:

_____ Does not demonstrate place value understanding
(represents each part of the number in units, such as "3" in 32 means "3" instead of "30" or"3 10s")

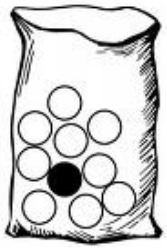
Describe child's thinking:

Task #2: Marble Task

Present loose cubes, cubes in towers of 10 and/or base-10 blocks, and paper and pencil.

Introduce marble context. Show pictures below, and say,
"At the toy store, marbles are packed into *small bags* of 10 marbles each. The *small bags* are packed into *boxes* of 10 bags each."

marble / canica



small bag / bolsita

box / caja

Problem A. If Elly has 5 bags of marbles, how many marbles does she have?

Numbers Used: _____ Child's Answer: _____

_____ Direct Models
- _____ by 1s
- _____ by 10s
  - ____ counts by 1s
  - ____ counts by 10s

_____ Does Not Solve

_____ Counting / Adding Strategy
- _____ skip counts by 10s and/or adds 10s (describe)
- _____ other (describe)

_____ Recalled Fact / Immediate Place Value Knowledge ("5 10s is 50")

Problem A (Harder): What if she has 14 bags? How many marbles would she have?

Numbers Used: _____ Child's Answer: _____

_____ Direct Models
- _____ by 1s
- _____ by 10s
  - ____ counts by 1s
  - ____ counts by 10s

_____ Does Not Solve

_____ Counting / Adding Strategy
- _____ skip counts by 10s and/or adds 10s (describe)
- _____ other (describe)

_____ Derived Facts
(10 10s is 100 plus 4 10s is 40)

_____ Recalled Fact / Immediate Place Value Knowledge

Problem B.

Hector has 7 bags of marbles, and 4 individual marbles. How many marbles does he have? *(Make sure child understand difference between bags, and individual marbles. Refer to the pictures as needed.)*

Easier Numbers (if needed): (2,3) **Harder Numbers (if needed):** (18, 5)

Numbers Used: _____ Child's Answer: _____

_____ Direct Models
- _____ by 1s
- _____ by 10s and 1s (groups tens and ones)
  - ____ counts by 1s
  - ____ counts by 10s and 1s

_____ Does Not Solve/Invalid strategy
(e.g., child adds 7+4, gets 11)

_____ Counting / Adding Strategy
- _____ counts by 10s and 1s (describe)
- _____ other (describe)

_____ Derived Facts
FACT _____

Handout CS 9
Problem Solving Interview: Base Ten Concepts 2

Goal

Your goal in this assessment is to find out what the student understands, without your assistance. If a child is having difficulty with a problem, you may change the numbers or move on to a new problem type. There is no need to show students how to solve a problem. If you feel the child needs some support, you can just say something like, " Really good try ____. We are going to work on this some more this year." Or " This is a hard problem, it is okay if you do not finish it right now."

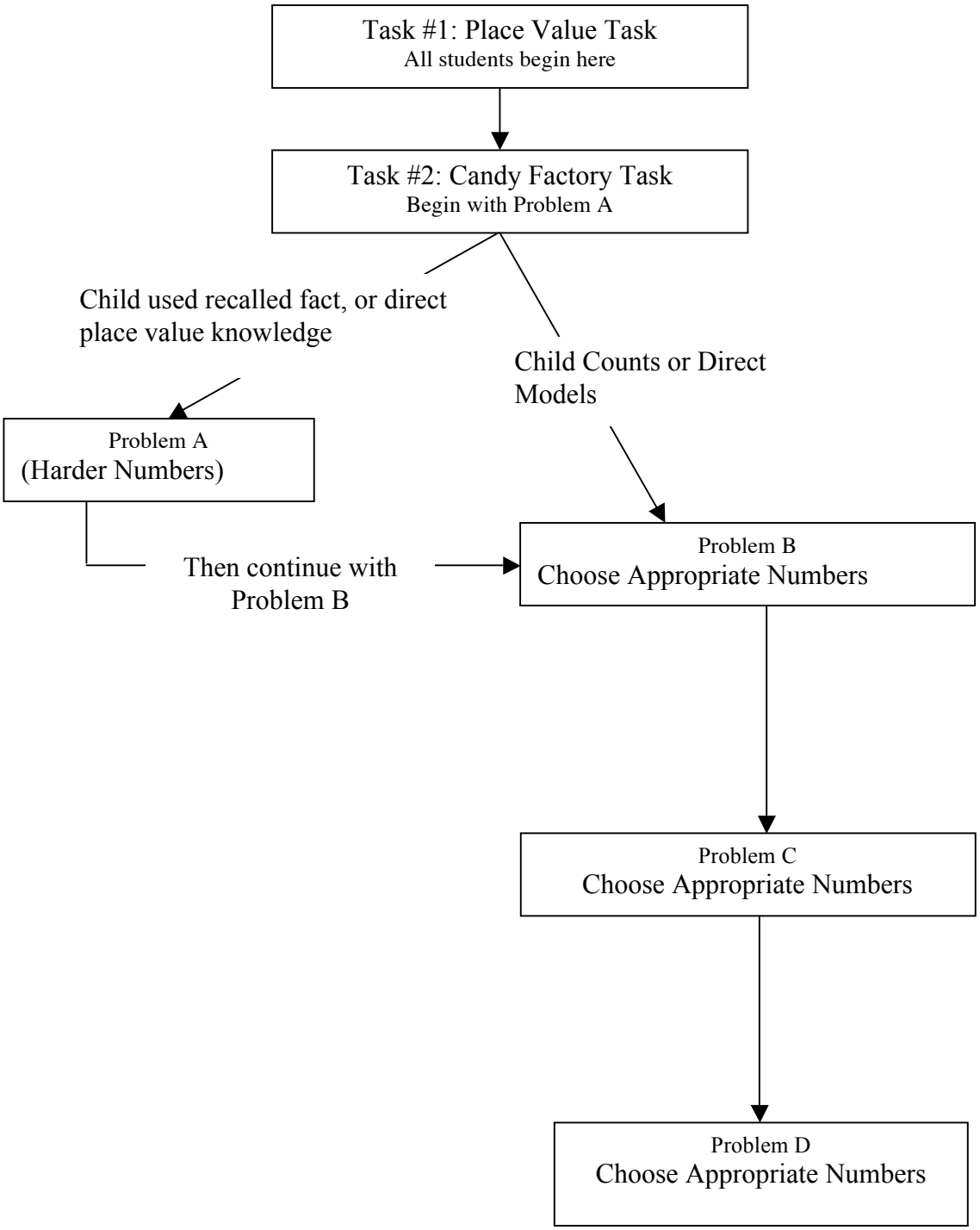
Problems

The Base Ten Concepts 2 interview includes 2 kinds of tasks. All children will begin with Task #1, which is a brief place value task. For Task #2, we made a decision tree to show which problems to do when, according to the strategies the child uses. Children will solve between 3 and 4 problems in this assessment. If a child at any grade level is having trouble, you could substitute easier numbers. If a child at any grade level is succeeding easily, you could try some of the problems with harder numbers.

Administration of the Assessment:

This assessment is designed for teachers to give one on one with students. Each assessment should take about 15 minutes on average since you will only give three to four problems for each assessment. This can be done in a corner of the classroom as other students are working independently (much as you might do with a reading record), or you can find another space such as a hallway or other area.

Base Ten Concepts Assessment: Decision Tree for Interview



Task #1: Place Value Task

Put a pile of individual cubes or counters (more than 26) on the table.

Show the number 26 written on a card.
Ask child to read the number.
Ask the child to show the number with cubes.

Then point to the 6 on the cards and ask "What does this part mean? Could you show me with the cubes what this part means?"

Similarly, point the 2 and ask "What does this part mean? Could you show me with the cubes what this part means?"

_____ Demonstrates place value understanding (the 2 represents 20 cubes, or 2 tens)

Describe child's thinking:

_____ Does not demonstrate place value understanding
(represents each part of the number in units, such as "2" in 26 means "2" instead of "20" or"2 10s")

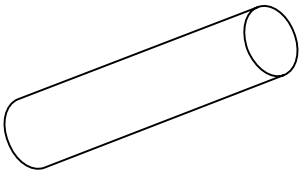
Describe child's thinking:

Task #2: Candy Factory Task

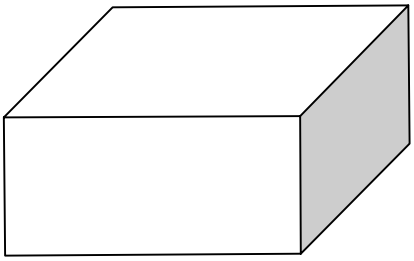
Present loose cubes, cubes in towers of 10 and/or base-10 blocks, and paper and pencil.

Introduce candy factory context. Show pictures below, and say,
"At the candy factory, candies are packed into *rolls* of 10 candies each. Rolls are packed into *boxes* of 10 rolls each."

candy / dulce



roll / rollo



box / caja

Problem A. If Alex has 4 rolls of candies, how many candies does she have?

Numbers Used: _____ Child's Answer: _____

_____ Direct Models
_____ by 1s
_____ by 10s
____ counts by 1s
____ counts by 10s

_____ Does Not Solve

_____ Counting / Adding Strategy
_____ skip counts by 10s

_____ other (describe)

_____ Recalled Fact / Immediate Place Value Knowledge ("4 10s is 40")
FACT _____

Problem A (Harder): What if she has 16 rolls? How many candies would she have?

Numbers Used: _____ Child's Answer: _____

_____ Direct Models
_____ by 1s
_____ by 10s
____ counts by 1s
____ counts by 10s

_____ Does Not Solve

_____ Counting / Adding Strategy
_____ skip counts by 10s and/or adds 10s (describe)

_____ Derived Facts
(10 10s is 100 and/or 6 10s is 60)

_____ Recalled Fact / Immediate Place Value Knowledge
(16 x 10 = 160; or 16 tens is 160)

Problem B.

Hannah has 6 rolls of candies, and 5 individual candies. How many candies does she have? *(Make sure child understand difference between rolls, and individual candies. Refer to the pictures as needed.)*

Easier Numbers (if needed): (2,5) **Harder Numbers (if needed):** (14, 5)

Numbers Used: _____

Child's Answer: _____

_____ Direct Models
_____ by 1s
_____ by 10s and 1s (groups tens and ones)

_____ Counting / Adding Strategy
_____ counts by 10s and 1s (describe)
_____ other (describe)

_____ Derived Facts
FACT _____

_____ Immediate Place Value Knowledge
("65, because 6 tens and 5 ones is 65")

_____ Does Not Solve/Invalid strategy
(e.g., child adds 5+6, gets 11)

Problem C. **Measurement Division Forming Groups of 10s and 1s**

It's Hannah's job to put the candies into rolls, with 10 candies in each roll. How many rolls can she make with 53 candies?

Easier Numbers: 23 candies
Harder Numbers: 123 candies, 275 candies, 1110 candies
(child should find how many boxes, and how many rolls)

Numbers Used: _____ Child's Answer: _____

_____ Direct Models
_____ by 1s
_____ by 10s and 1s (groups 10s and 1s)
_____ (other)

_____ Counting / Adding Strategy
_____ Counts by 10s (describe)
_____ other (describe)

_____ Derived Facts
FACT _____

_____ Immediate Place Value Knowledge
(e.g., "5 rolls, because there are 5 tens in 50")

_____ Does Not Solve / Invalid strategy

Problem D. Number Composition

Hannah's brother Isaac comes to the store to buy candies. He wants to by 68 candies. He can buy rolls of 10 candies, and he can buy individual candies. How can he do it? How many rolls should he buy, and how many individual candies? Can you tell me different ways he could buy 68 candies?

Easier Numbers: 35 candies

First way:

Second way:

Third way:

Handout CS 10
Interview Scenarios

Note: These scenarios are based on scenarios developed by Susan Empson.

Here is a list of things that could happen when you are interviewing a child, and/or interacting with a child in your class. What would you do if they happened?

1. You ask a child the problem: "Sandie had 4 cats. Anne Marie gave her 9 more cats. How many cats does Sandie have now?" The child puts out 4 cubes, pauses, moves his lips, and then says 13. You ask him how he solved it and he says, "I added".

2. A child solves all the problems you give her using either derived facts or recall.

3. You ask a child the following problem: "Sara has 12 baseball caps. José has 8 baseball caps. How many more baseball caps does Sara have than José?" He can not solve the problem.

4. You ask a child the following problem: "Becky has 5 cupcakes. Christine gives her some more cupcakes. Now Becky has 16 cupcakes. How many cupcakes did Christine give Becky?" The child puts out 5 cubes and then puts out 16 more cubes pushes them together, counts them all and says 21.

5. A child takes about a minute to solve a problem. He gets the correct answer without counters. When you ask him how he got that answer he says, "I just knew it."

6. You ask a child the following problem: "Nina has 7 goldfish. She wants to buy 8 more goldfish. How many fish would she have then?" She cannot solve the problem.

7. A child solves a problem using a traditional algorithm for addition, subtraction, multiplication or division, and gets the correct answer. What do you do?

8. A child uses a “buggy” algorithm to solve a problem, and arrives at an incorrect answer. What do you do?

9. What if the child explains their strategy for solving a problem. And you can’t follow their reasoning? What do you do?

10. What if a child uses a valid strategy to solve problem but makes a minor counting error? What do you do?

Handout CS 11
Assignment and Write up: Problem Solving Interview

The purpose of this assignment is to assess one child's understanding of problem solving. Keep in mind that you are assessing the child's *understanding,* rather than his or her accuracy at solving problems (i.e., you will be going beyond simply assessing right and wrong answers).

Your goal in this assessment is to find out what the student understands, without your assistance. If a child is having difficulty with a problem, you may change the numbers or move on to a new problem type. There is no need to show students how to solve a problem.

You will implement an interview protocol including a variety of whole number problem types. You will ask the child six to eight problems. **Further information about which and how many problems to pose will be discussed in class.** You will use these problems to assess your case study child's mathematical understanding. In a written reflection of this interview, you will describe what the child understands, include evidence to support your conclusions, and reflect upon the information you gained and where you would go from here.

CONDUCTING THE INTERVEIW/ASSESSMENT
For each question,

1) Read the problem to the child. (Re-read as many times as the child needs. Read the entire problem each time. Do not read it in parts.) It is fine to have the problem written out for the child, but in this case, also read the problem to the child and let them follow along.
2) Allow the child to use manipulatives, their fingers, paper and pencil, a mental strategy, or anything else they would like to solve the problem.
3) Ask the child, *"How did you figure that out?"* when he or she is finished solving the problem. Probe the child's thinking until you understand exactly how he or she solved the problem. (Respond to incorrect answers in the same way.)

Organize your report into the sections listed below.
REPORT OF THE INTERVIEW/ASSESSMENT
Part 1. Summary of interview
Provide the following information for each problem that you gave the child

a. the problem and problem type
b. the numbers you gave the child for each problem and the child's answer

(In writing up the Addition/Subtraction and Multiplication/Division interviews, use the space entitled "Description of Strategy" on Handout CS4, "Addition/Subtraction and Multiplication/Division Problems.")

Using the notes you took while conducting the interview, write a clear and *specific* description of what the child did including the strategy used. Write as completely as possible what transpired as the child solved the problem. This includes not only the final solution path, but also what led up to it including other attempts as solving by the child as well as conversation between you and the child, prompts you gave and so forth. At the end of each description, indicate what strategy or strategies the child used to solve the problem (direct modeling, counting, numeric).

Note: Describe *as specifically as possible* what the child said and did. For example, do not only say, "The child used cubes to solve the problem." You need to explain *how* the child used the cubes. For 13 – 6, you might say, "The child counted out 13 cubes by 1, connecting each of the cubes into a long train. Then he counted 6 cubes (by twos) from the top of the train and broke off those 6 cubes. He put the 6 cubes aside and counted (by ones) the number of cubes that were still in the train and found that there were 7 cubes left. He answered the problem by saying that he had 7 cookies left."

If I child used multiplication do not only say, "The child multiplied to get the answer." You will need to explain how she multiplied. If she used the standard algorithm, say that. If she used a partial product approach, say that and discuss how she initiated the strategy (e.g. which number did she start with? How did she record her algorithm, what did she write down?). If she multiplied in her head, explain what she did (which means you need to have asked what she did). If she directly modeled, describe what she did with the materials she used to arrive at the solution.

Part 2. Analysis of problem solving
Next, choose one or two problems that the child solved, (NOTE: for PSTs completing one write up of multiple interviews, choose ONE problem that seems interesting from each interview) and in approximately one to two pages provide an analysis of what you think the strategy reveals about the child's understanding of number and the arithmetic operation(s) involved. Does the child directly model the solution? Does the child use a counting strategy? Consider the level of abstraction the strategy demonstrates. For example: Does the student understand the commutative property or the relationship between addition and subtraction? Can the child use known number facts to derive other facts? Does the child demonstrate knowledge of tens? Does the child demonstrate operation sense? Etc…

Part 3. Summary reflection on interview: What did you learn about the student's understanding and yourself as an interviewer from your problem solving interview

- What did you learn about the child's mathematical understandings, and what happened during the interview that led you to those conclusions? What evidence do you have that the student was thinking conceptually and/or procedurally? What evidence do you have of student confusions, misconceptions, and/or partial understandings? Did the student ask questions? What did the student's questions reveal about his/her knowledge? Did the student use more than one strategy? Did they abandon a strategy? How persistent was the student in trying different strategies?
- Include a short reflection about what you learned about yourself as an interviewer. Reflect on your interactions with the child: How well did you elicit your students thinking? What kinds of questions did you ask? What kinds of comments did you make? To what extent did your questions and/or comments help you to learn more about your students understanding?
- Link your understandings to one or more of the class readings

Please limit this Part 3 reflection to 1-3 pages. Please structure your reflection by answering the questions posed above. If you like, you can use the questions as headings in your reflection.

A note of caution about your write-ups:

- Please do not include an assessment of the child's overall personality (e.g., happy, self confident or nervous child) or overall ability level (e.g., smart or slow child). You may want to comment on a child's performance solving a particular problem (e.g., child was confident about her solution strategy to problem x) but please avoid generalities as you will only meet with the child for about 30 minutes.
- In your write-ups, please use only the **child's first name** in order to preserve his/her anonymity.

Activity 4: Synthesizing and Connecting Across Case Study Activities (Written Assignment and Mock Parent-Teacher Conferences)

Description of Synthesizing and Connecting Assignment
In a report written outside of class, PSTs elaborate on what they have learned from the previous Case Study Activities. For this assignment, PSTs draw on all they have learned about their case study students from the various activities and interactions with their student throughout the semester. PSTs will use what they learned to plan for the future. This activity can be completed as a written assignment alone, submitted at the end of the semester (or as a portion of a final exam), or it can be used in combination with a *mock parent-teacher conference* activity (an in-class activity for PSTs to share their learning with a peer). The written assignment instructions are provided in Handout CS 12. If an instructor opts to follow-up the written assignment with mock parent teacher conferences, refer to the Lesson Outline below.

Goals for Synthesizing and Connecting Across Case Study Activities
For the PSTs to synthesize the information and understandings they have learned across the activities, make connections among students' MMKB, and generate ideas for future teaching and learning based on what they know about their case study student. For PSTs to consider, rehearse, and reflect on ways to communicate and build relationships with parents in support of students' learning.

Lesson Outline for Mock Parent-Teacher Conferences

LAUNCH: After students have written the report described above, students should bring their report and student work to class as background information to use in talking with parents/family for a "Mock Parent Conference." In order to interrupt the typical school-home dynamic that positions teachers as the "knowers" and parents as the "learners" of their children, PSTs need the "Getting to Know You" interviews available so that the PST playing the parent can read about the child, bring that knowledge about the case student to the conference, and prepare them to respond to Prompt #4 below.

The instructor provides a brief overview for how the conferences will be conducted, explaining that in pairs, each PST will take turns in roles as a teacher and as a parent/family member for PSTs case study students. Mirroring the amount of time usually spent discussing mathematics in conferences, pairs will spend approximately 5 minutes discussing each student (PSTs stay in each role for approximately 5 minutes). The instructor should announce when 5 minutes are up – time for PSTs to switch roles. After each PST has served in each roll, the pair provides feedback to each other, as indicated on the prompts.

To prompt PSTs about issues and considerations in communicating with parents, instructors briefly discuss the following. These ideas are intended to open up thinking and can be revisited in more depth in the summary, but we have found that introducing these ideas prior to conferencing is helpful to preemptively address ideas that could come up in pair discussions:

- Ask PSTs if they have attended conferences as a parent or family member. If so, ask PSTs' to share their experiences and feelings. If PSTs have had these experiences, often they have been difficult or painful – with stories of how teachers offended parents by

focusing only on behavioral problems or what students did not know. If PSTs have not had these experiences, ask them to imagine how they would want to experience a conference about their child. These comments can be used to discuss the importance of focusing on strengths and next steps for learning, rather than focusing on deficits for children.

- A common perspective we have heard is that "some parents do not care." It is helpful to state up front that parents care about their children, even if parents are not behaving in ways that schools often expect. Instructors can ask PSTs to consider ways they can broaden their perspectives on caring, obligations teachers have for meeting students needs (rather than blaming parents), and ways we can build positive relationships and communication with parents.

Next, make sure that pairs have space to talk and instruct pairs to use the prompts below to guide their mock conferences.

EXPLORE:

PSTs conference in pairs, using the prompts below to structure their conversation. The instructor announces time after 5 minutes (switch roles), 10 minutes (debrief), and 15 minutes (come together as a whole class for the Summary).

Prompts for Mock Parent Conferences (in pairs, each taking a turn as Teacher and as Parent):

Teacher:

1. Summarize students' competencies, evidence of learning, and ideas for next steps.
2. Support your findings with evidence to illustrate your ideas so parents understand.
3. Show your "problem" and use it to explain how you intend to support the students' learning.
4. Ask the parents/family questions that might help you to understand their child and support their child's learning. Focus on learning more about a child's competencies from the perspective of the parents/family.

Parent:

1. Ask questions, offer comments, etc.
2. After the "conference" is over, provide feedback to the teacher on how you felt, how well you understood the teacher's points, how well the findings were evidenced.

SUMMARY:

After pairs have each had a turn in each role and shared feedback, instructors facilitate a whole class discussion about the conferences. As appropriate, return to the initial discussion about any deficit perspectives or past negative experiences that were shared during the Launch.

Prompts for Whole Group Discussion (these prompts can be used for whole group discussion and/or written reflections after Mock Parent Conferences):

1. What went well in your conversation? What was difficult or challenging? What would you want to phrase or communicate differently?
2. What are some key ideas you want to keep in mind when communicating with families with the goal of building relationships with family and supporting students' learning?
3. What kind of information can you learn from parents and families that will help you to support students' learning?

Additional Resources for PSTs and Instructors for Synthesizing and Connecting Across Case Study Activities:

Note: With the exception of Ramirez & Celedón-Pattichis, the following readings are helpful for PSTs to read to learn more about building relationships and interacting with parents. The Ramirez & Celedón-Pattichis chapter focuses on instructional implications for working with ELLs in mathematics.

Aguirre, J., Mayfield-Ingram, K., Martin, D. (2013). Routine practices to engage parents in promoting positive math learning and identity. In *The impact of identity in K-8 mathematics: Rethinking equity-based practices* (pp. 87-103). Reston, VA: National Council of Teachers of Mathematics.

Civil, M & Menendez, J. M. (2010) *Involving Latina and Latino parents in their children's mathematics education*. NCTM Research Brief. http://www.nctm.org/news/content.aspx?id=27539

Jackson, K. & Remillard, J. (2005) Rethinking parent involvement: African American mothers construct their roles in mathematics education of their children. *School Community Journal*. *15*(1) 52-71.

Kozol, J. (2007). Reaching out to get to know the parents of our children. In J. Kozol (Ed.), *Letters to a young teacher* (pp. 21-32). New York, NY: Crown.

Peressini, D. (1997). Building bridges between diverse families and mathematics classrooms: Parent involvement in mathematics education. In M. M. Trentacosta (Ed*.), Multicultural and gender equity in the mathematics classroom: The gift of diversity* (pp. 222-229). Reston, VA: National Council of Teachers of Mathematics.

Ramirez, N. G., & Celedón-Pattichis, S. (2012). Second language development and implications for the mathematics classroom. In S. Celedón-Pattichis & N. Ramirez (Eds.) *Beyond good teaching: Advancing mathematics education for ELLs* (pp. 19-37). Reston, VA: National Council of Teachers of Mathematics.

Handout CS 12
Synthesizing and Connecting Across Case Study Activities: Written Assignment Instructions for PSTs

A. Introduce your student: Include age, grade, school, gender, race/ethnicity, family background, home language(s), etc. Provide a summary of what you know about the student's background. If your student is an English language learner or a bilingual or multi-lingual student, include description of student's stage of language development for home language(s) and English, considering reading, writing, speaking and listening competencies and needs. Explain why you selected this student and how this student is different from you (compare/contrast the students' background with yours). This part is intended to be a brief introduction only.

B. Problem Design. Based on what you learned from all of your interactions, observations, and interviews with this child, what do you think would be a reasonable "next step" for the child in terms of mathematics instruction? More specifically, if you were to work with this child again, what **specific problem (or set of problems)** would you like to ask next to further this child's understandings? Note that it is not sufficient to only suggest giving the child more problems. Instead, you should decide what would be the next specific problem (including particular numbers, problem structures/types, and contexts) that you would like that child to solve. Be sure to include an explanation of what you hope to learn by posing that specific problem, and WHY this would be a reasonable problem to give the child (based on specific evidence you have gathered). Your justification should be based on what you learned about the child across ALL the multiple observations and interviews.

C. Instructional Implications & Conversation with Parents. For this part, reflect more broadly on how you can use what you learned from your interactions, observations, interviews with your case study student, and community exploration to inform your mathematics instruction. Think about preparing to have a conversation/conference with the case study student's parent(s) or guardian, in which you will share with the parent specific examples of what you have learned about their child, and how you plan to use what you have learned to inform your mathematics instruction. Your written reflection does not have to be an actual letter to the parents, but rather a written reflection that you might use to prepare for and support your conversation with the child's parents. Include the following:

1. Be specific as you discuss the pedagogical/instructional implications of what you learned about your case study student. Your response MUST include explicit examples from the interviews and observations you conducted ("Getting to know your student" interview, shadowing and observing your student in multiple contexts, problem solving interviews on whole number operations) to support and justify your claims, teaching recommendations, and next steps. It is not enough to say things like, "I would give the student more math problems/practice" or "I would make the math more fun/interesting" or "I would have the student work in groups." You must give specific reasons grounded in your student's case. Explicit grounding in and connections to course readings and research (with citations) are also expected.
2. Discuss what you learned about the student that you could draw upon in math lessons. Discuss how you would set up/design mathematics instruction in your classroom to build

upon your students' competencies/strengths and experiences (including out-of-school experiences). Are there things that you learned that would influence how you structure lessons for this student? For example, based on what you learned about your student, what kinds of participation structures and/or instructional support would be useful? What kinds of peer interactions would be helpful? What kinds of family and/or community connections/activities/contexts would be helpful to support this student's math learning?

3. Are there things you learned that you could connect to during a math activity? Can you think of other (beyond Part B) math projects or investigations that would really allow your student to demonstrate the student's competence, understandings, and/or skills?
4. Describe any remaining questions you have about a child. What more would you like to know? What might you ask parents/family so that you could learn more?

References

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999) *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Empson, S., Junk, D. & Turner, E. (2006). *Formative mathematics assessment for use in grades K-3*. Unpublished Assessments.

Ginsburg, H. (1997). *Entering the child's mind: The clinical interview in psychological research and practice.* New York, NY: Cambridge Press.

Jacobs, V. & Ambrose, R. (2008). Making the most of story problems. *Teaching Children Mathematics* 15(5), 260-266.

Phillipp, R., Cabral, C., & Schappelle, B. (2012). Integrating mathematics and pedagogy (IMAP): Searchable collection of children's-mathematical-thinking video clips. Boston, MA: Allyn & Bacon.

Spindler, G., & Spindler, L. (1982). Roger Harker and Schonhausen: From the familiar to the strange and back again. In G. Spindler (Ed.), *Doing the ethnography of schooling* (pp. 20–46). Prospect Heights, IL: Waveland.